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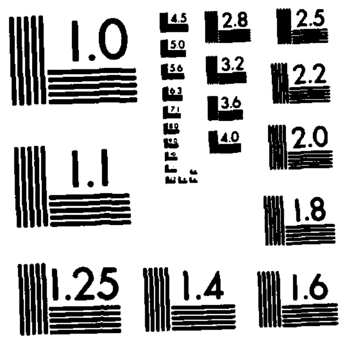

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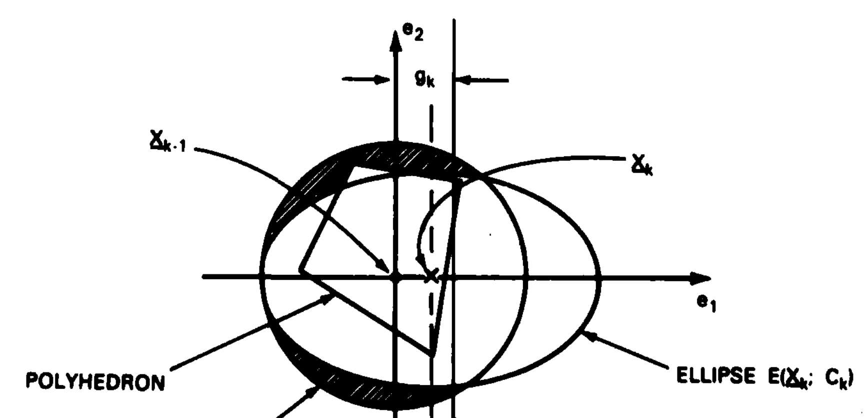

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

NON-MODEL BASED ESTIMATION WITH APPLICATIONS TO SENSOR CALIBRATION USING MULTIPLE SENSORS

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Group 33

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ABSTRACT

A non-model based estimation procedure is introduced. The best estimate is defined to be the geometric center of the intersected error volume, and the error of the best estimate is defined as the distance of the intersected error volume measured from the center. Khachian's algorithm is extended to find the best estimate. When the algorithm is viewed as a system of nonlinear difference equations, conditions are established to test the existence of periodic solutions. A stopping rule is also introduced. A strategy for finding the error of the best estimate is described. Examples are given to illustrate the estimation procedure.

1. INTRODUCTION

The goodness of an estimator is assessed conventionally by both the bias and variance of the estimator. The true meaning of the bias and variance of an estimator can not be fully grasped unless the underlining model is accurately defined. Perhaps, this is the reason why model reliability first introduced by Akaike [1] has been widely accepted as an additional quantity for assessing an estimator.

In reality, computability of bias, variance and model reliability of an estimator is a serious question especially when the underlining model is complicated. Some qualitative answers to this computability question have been documented in [2], [3], and [4]. In particular, it has been shown in [3] that observability is a necessary condition for the existence of an unbiased estimate with bounded variance. Shall we give up if we find that the system is not observable and there is no way to compute the bias of the estimator? I would predict that most people would not give up based on the evidence that abundant Monte-Carlo simulation results have been reported in various literatures. We certainly pay little attention to the simulation results unless the model used is truly reliable that in turn, can hardly be confirmed through simulations.

In principle, a-priori information can be used to improve the quality of an estimator. At least three different incidents described below prove on the contrary that a-priori information actually hurts.

(i) The a-priori information is biased,

(ii) It is used improperly,

(iii) The model used is not reliable.

An example of case (ii) mentionded above has been examined in [4] for a nonlinear initial state estimation problem.

To put the above abstract description of the difficulty of a conventional statistical estimation procedure into perspective, we shall confine ourself to the context of sensor calibration and trajectory estimation problems from this point on. The proposed estimation procedure reported herein certainly can be applied to various other problems as well. All examples given are related to sensors along the Western Test Range. Furthermore, sensor biases and calibration constants are treated as interchangeable terminologies.

In calibrating a sensor, the first step is to estimate the position as a function of time of a target. The target can be an inertial star, a satellite, a re-entry vehicle, or a calibration balloon etc. that can be tracked by calibrated sensors and the sensor to be calibrated. Treating calibration constants as unknowns, the calibration procedure can then be cast into an inversion problem that relates the position of the target to the sensor tracking data. When some calibration constants are not observable, the inversion problem does not have a unique solution. Examples of these unobservable cases can be found in [4] and [5].

The hypothetical scenario described as follows motivates the study of non-model based estimation reported herein. A newly designed radar system in the island of Roi-Namur needs to be calibrated. A calibration balloon is launched and drifted across the Kwajalein lagoon. There are one calibrated radar, and five calibrated optical sensors as well as the new radar system along the Kwajalein atoll that track the calibration balloon. The motion model of a balloon certainly is not reliable. Observability of calibration constants of the new radar is questionable. Furthermore, for each calibrated sensor, a-priori information is available but no guarantee about its authenticity. The problem to be addressed is how to find a way to calibrate the new radar based on all information available.

For simplicity, we assume that the calibration constants are in terms of range, azimuth, and elevation biases.* The calibration problem posed above is therefore reduced to estimating the position of the target at a fixed time instant using all available information.

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* Actual calibration procedures are much more complicated than the assumption made here. Therefore, more than one point in the space is needed.

For each calibrated sensor, there is a best position estimate with an error volume of the shape shown in Fig. 1, at the fixed time based on all information available to the sensor.

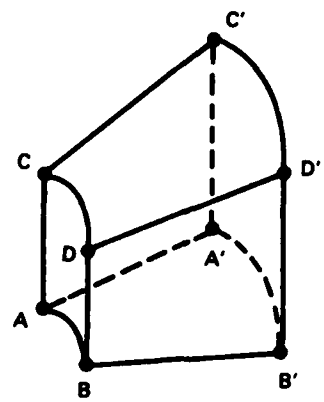

Fig. 1. The shape of an error volume.

Thus, we have five error volumes from optical sensors and one error volume from the calibrated radar in the space. These error volumes may or may not intersect. When there exist non-intersecting error volumes, subjective opinions arise. To maintain the argument as objective as possible, we shall assume that the majority of the error volumes do intersect. We shall define the position estimate of the target as the geometric center of the intersected error volume, and the error of the position estimate is defined as the distance of the intersected error volume measured from the geometric center. In the sequel, a rigorous mathematical treatment of the above description will be formulated. A methodology of finding the position estimate and its error is also described. Finally, examples are given.

2. MATHEMATICAL FORMULATION

2.1. Description of a Polyhedron

The error volume of each sensor is shown in Fig. 1. For mathematical tractability, we assume that the arcs AB (A'B') and CD (C'D') are straight lines. Thus, the shape becomes a prismatoid. The vertices of each prismatiod are given. A polyhedron is formed by intersecting a number of prismatoids. Following a consistent convention, the faces of a prismatoid can be represented by a number of linear equations derived from the given vertices. The polyhedron can therefore be described by a matrix inequality given by

$$A \underline{x} \leq \underline{b} \tag{2.1}$$

where $\underline{x}$ = a 3 by 1 vector,

A = a 6n by 3 matrix, where n is the number of prismatoids,

$\underline{b}$ = a 6n by 1 vector whose components are non-negative.

The i^{th} row of A and $\underline{b}$ will be denoted by A_i and b_i respectively. The transpose of a matrix A is denoted by A^T. The Euclidean norm of a vector is denoted by $||\cdot||$.

When there are prismatoids that do not intersect with the rest of the prismatoids, contradicting inequalities exist in (2.1). Redundant inequalities may also exist in (2.1) if one prismatoid contains the other. Algorithms are available for detecting both contradicting and redundant inequalities [6], [7].

The algorithm introduced in [7] will be used.

2.2 Khachian's Algorithm

Beginning with a point outside the polyhedron given by (2.1), Khachian's algorithm can be used to find a point inside the polyhedron. The basic method that Khachian follows is to construct a minimum ellipsiod that contains one portion of another ellipsoid cut by a hyperplane. An ellipsoid, E (x; C), can be represented by its center $\underline{x}$ and a positive-definite matrix C given by

$$E(\underline{x};\ C) = \{\underline{y};\ \underline{y} = \underline{x} + C\underline{z},\ ||z|| \leq 1\} \tag{2.2}$$

Initially, let $\underline{x}_0$ be outside the polyhedron given by (2.1), and $E(x_0;\ C_0)$ be the ellipsiod so large that it contains the polyhedron. The algorithm goes as follows.

$$\begin{aligned}
\underline{x}_k &= \underline{x}_{k-1} - \frac{C_{k-1} A_k^T}{\sqrt{A_k C_{k-1} A_k^T}} \alpha_k \\
C_k &= \beta_k \left[C_{k-1} - \gamma_k \frac{(C_{k-1} A_k^T)(C_{k-1} A_k^T)^T}{A_k C_{k-1} A^T_k} \right] \\
\alpha_k &= \frac{1 + m g_k}{m + 1} \\
\beta_k &= \frac{(1 - g_k^2) m^2}{m^2 - 1} \\
\gamma_k &= \frac{2\alpha_k}{1 + g_k} \\
g_k &= (A_k \underline{x}_{k-1} - b_k) \Big/ \sqrt{A_k C_{k-1} A_k^T}
\end{aligned} \tag{2.3}$$

where m = the dimension of $\underline{x}_k$, in our case m = 3.

The volume of the ellipsoid is proportional to its determinant (abbreviated by Det). A recursive relationship for the determinants is given by

$$\log(\mathrm{Det}\ C_k) = \log(\mathrm{Det}\ C_{k-1}) + m \log \beta_k + \log \left[\frac{(m-1)(1-g_k)}{(m+1)(1+g_k)} \right] \quad (2.4)$$

The parameter g_k measures the distance from the previous center $\underline{x}_{k-1}$ to the hyperplane $A_k \underline{x} = b_k$.

Several important situations are summarized as follows.

(i) If $|g_k| > 1$ then the hyperplane $A_k \underline{x} = b_k$ does not intersect the polyhedron. Therefore, the corresponding inequality is contradicting to others.

(ii) If $|g_k| \leq 1$ and $\underline{x}_{k-1}$ is outside the hyperplane $A_k\underline{x} = b_k$, then $g_k > 0$.

(iii) If $\underline{x}_{k-1}$ is inside the hyperplane $A_k\underline{x} = b_k$ then $g_k < 0$. If $g_k < 0$ but $\underline{x}_{k-1}$ is outside the polyhedron then $A_k \underline{x} \leq b_k$ is a redundant inequality.

Abundant literatures are available for detailed descriptions of Khachian's algorithm.

Suppose that a point inside the polyhedron is found. The point need not be the geometric center of the polyhedron. Since the shape of the polyhedron can be very irregular, its geometric center is tough to define. In the next section, we shall see that extending Khachian's algorithm into the polyhedron can arrive at a satisfactory definition of the geometric center of a polyhedron.

3. BEST ESTIMATE AND ITS ERROR

3.1 Existence of Periodic Solutions

Let $\underline{x}_0$ be the first point found inside the polyhedron and $E(\underline{x}_0; C_0)$ be the corresponding ellipsoid containing the polyhedron. Khachian's algorithm ensures that all $\underline{x}_k$ will remain inside the polyhedron. However, the ellipsoid $E(\underline{x}_k; C_k)$ does not necessarily contain $E(\underline{x}_{k-1}; C_{k-1})$. From now on, we shall treat (2.3) as a system of difference equations with initial condition $(\underline{x}_0, C_0)$, and call $(\underline{x}_k, C_k)$ as the solution of the system. Furthermore, we shall assume that all inequalities in (2.1) are compatible and relevant. The number of inequalities is assumed to be N. After k reaches N, the first inequality is called upon. We assign a new index N+1 to the first inequality and count on. Therefore, A_k is periodic with period N. The following theorem establishes the situation for the existence of a periodic solution.

<u>Therem 3.1</u> If there exists K such that both conditions described below satisty then $(\underline{x}_{k+N}, C_{k+N}) = (\underline{x}_k, C_k)$ for all $k \geq K$.

(i) $K \geq N$, $\text{Det } C_K = \text{Det } C_{K-N}$ (3.1)

(ii) Either $C_K \geq C_{K-N}$ or $C_K \leq C_{K-N}$

<u>Proof</u>: Since both C_K and C_{K-N} are positive definite, Minkowski's inequality [8] is applied to obtain

$$\text{Det}\left[C_K + C_{K-N}\right] \geq \text{Det } C_K + \text{Det } C_{K-N} \tag{3.2}$$

and equality attains if $C_K = \lambda\ C_{K-N}$ where λ is a non-negative scalar. Based on (3.1), (3.2) and results in [9], we conclude that $C_K = C_{K-N}$. Therefore, $g_{K+1} = g_{K-N+1}$, that in turn, implies

$$\alpha_{K+1} = \alpha_{K-N+1}$$

$$\beta_{K+1} = \beta_{K-N+1}$$

and

$$\gamma_{K+1} = \gamma_{K-N+1}.$$

Thus, C_k is periodic for all $k \geq K$. Since A_k is periodic with period N, $\underline{x}_k$ is also periodic for all $k \geq N$.

When a periodic solution exists, the polyhedron is symetric with repect to its center. The best estimate denoted by $\hat{\underline{x}}$ is given by

$$\hat{\underline{x}} = \frac{1}{N} \sum_{k=K+1}^{N} \underline{x}_k \tag{3.3}$$

Based on many empirical results, approximate periodic solutions do exist. However, it seems to be beyond my ability to prove it theoretically. The remainder of this subsection may be skipped for those who believe the statement made above.

We shall now develop a criterion that can be used to detect the inequality that when used will move the previous center of the ellipsoid away from the center of the polyhedron.

Theorem 3.2 If $g_k < -1/m$ then $E(\underline{x}_k; C_k)$ does not contain $E(\underline{x}_{k-1}; C_{k-1})$.

Proof: Since C_{k-1} is positive-definite, it can be decomposed into the form given by

$$C_{k-1}^{-1} = D^T D$$

where D is a nonsingular matrix. Let F be an orthogonal matrix such that

$$F \quad \frac{D^T A_k{}^T}{\left|\left|D^T A_k{}^T\right|\right|} = \underline{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Define an affine transformation T such that

$$\begin{aligned} \underline{z} &= T(\underline{x}) \\ &= F D (\underline{x} - \underline{x}_{k-1}). \end{aligned}$$

In z-domain, $E(\underline{x}_{k-1}; C_{k-1})$ becomes a unit sphere centered at the origin. The sphere projected on the plane of $\underline{e}_1 - \underline{e}_2$ is shown in Fig. 2. The hyperplane $A_k\underline{x} = b_k$ projected on the same plane is also shown in Fig. 2. Note that α_k is the distance from $\underline{x}_{k-1}$ to $\underline{x}_k$, $|g_k|$ is the distance from $\underline{x}_{k-1}$ to the hyperplane $A_k\underline{x} = b_k$, and $(\beta_k)^{-1/2}$ is the length of the minor axis of $E(\underline{x}_k; C_k)$. If $g_k < -1/m$ then $\underline{x}_k$ is located in between $\underline{x}_{k-1}$ and hyperplane $A_k\underline{x} = b_k$. Further-more, we have $(\beta_k)^{-1/2} < 1$. Thus, $E(\underline{x}_k; C_k)$ in z-domain cuts the unit sphere.

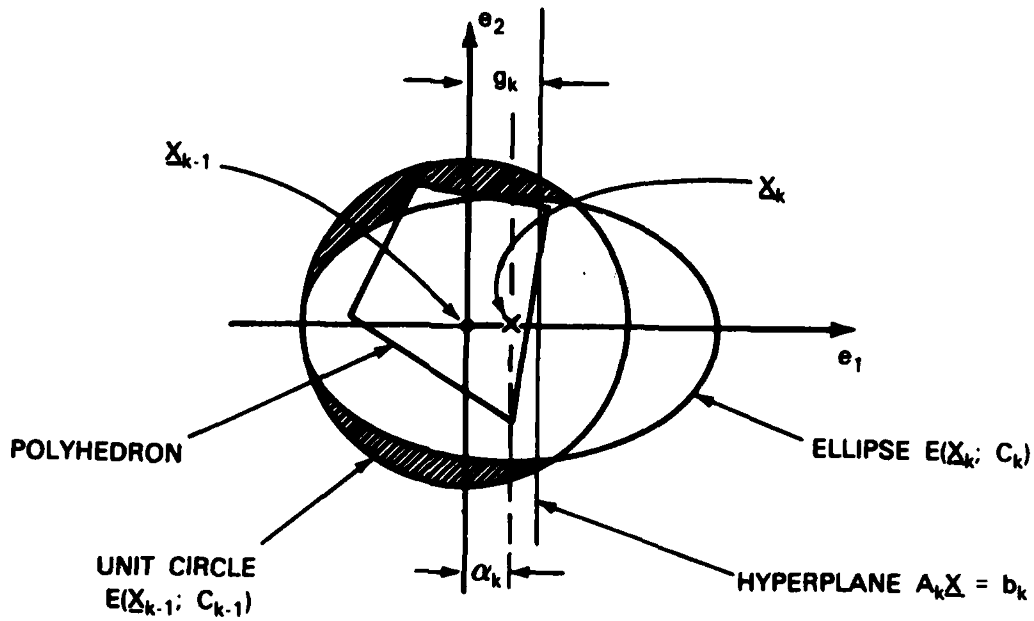

Fig. 2. Two-dimensional Khachian Diagram.

It is of interests to note that both E $(\underline{x}_{k-1}; C_{k-1})$ and E $(\underline{x}_k; C_k)$ have the same volumes if $g_k = -1/m$. After the algorithm is applied for a sufficiently large number of times, the center of the ellipsoid tends to move towards the center of the polyhedron. If $\underline{x}_{k-1}$ is already in the vicinity of the center of the polyhedron, then it is clear from Fig. 2 that applying $A_k\underline{x}_k \leq b_k$ moves $\underline{x}_{k-1}$ away from the center in $\underline{e}_1 - \underline{e}_2$ plane. Thus, the decision criterion goes as follows.

(i) If $g_k < -1/m$ then skip $A_k\underline{x} \leq b_k$.

(ii) If for all inequalities $g_k < -1/m$ then $\hat{\underline{x}} = \underline{x}_{k-1}$.

Again, empirical results show that the above decision rule terminates the interation fairly quickly. Theoretical justification remains to be seen.

3.2 Error of the Best Estimate

Suppose, now that the best estimate $\hat{\underline{x}}$ is already found. A simple translation can move the origin of the coordinate system to $\hat{\underline{x}}$. We want to find a farthest point inside the polyhedron measured from the origin. Mathematically, the problem is given by

$$\text{maximizing } ||x|| \tag{3.4}$$

subject to (2.1).

Therefore, it is a convex maximization problem. One of the vertices of the polyhedron is a solution. It is clear that only a finite number of searching steps is required. In this subsection, a searching strategy is described for m=3.

Let $\underline{V}_0$ be a solution of (3.4) and let $\underline{V}_{ij}$ denote the i^{th} vertex of the j^{th} prismatoid. For each j, let $\underline{V}_j$ denote a vertex among $\underline{V}_{ij}$, i = 1, 2, ..., 8 that has a maximum norm. Furthermore, let $\underline{V}$ denote a vertex that has a minimum norm among $\underline{v}_j$, i = 1, ..., n. It is clear that we have the following inequality,

$$||\underline{V}_0|| \leq ||\underline{V}|| \tag{3.5}$$

There are three inequalities in (2.1) that are associated with the vertex $\underline{V}$. Except these three inequalities, those other inequalities that satisfy the inequality described below are redundant, and they can be discarded.

$$0 \leq A_i \underline{V} \leq b_i \tag{3.6}$$

To prove the above statement, let's suppose that $\underline{V}_0$ lies on the hyperplane given by $A_i\underline{x} = b_i$. By (3.6) we know that $\underline{V}$ lies in between the hyperplane and the origin. Therefore, we have

$$||\underline{V}|| \leq ||\underline{V}_0|| \tag{3.7}$$

Combining (3.6) and (3.7), we conclude that $\underline{V}$ is also a solution. Therefore, the inequalities satisfying (3.6) are redundant, and they can be discarded.

Selecting one relevant inequality that is not from a referenced prismatoid that is chosen arbitrarily, and two inequalities from the referenced prismatoid, we form a set of three simultaneous linear equations in order to find a new vertex. Two different situations may happen that are described below.

Case 1 The hyperplane associated with the first inequality is parallel to the edge formed by intersecting two faces of the referenced prismatoid. We, therefore, skip this inequality because no unique vertex can be found from this set of linear equations.

Case 2 The set of three simultaneous linear equations has a unique solution denoted by $\underline{U}$. A decision is made based upon the following rule.

Rule If $||\underline{U}|| \geq ||\underline{V}||$ then skip the first inequality mentioned above because $\underline{U}$ can not be a solution. If $||\underline{U}|| < ||\underline{V}||$ then put $\underline{U}$ in a pool of vertices including those of the referenced prismatoid, and proceed to select another set of three simultaneous linear equations. The new set of linear equations also contains one relevant inequality that is not from the referenced prismatoid, and two inequalities from the referenced prismatoid.

The same procedure repeats until each prismatoid has been chosen as the referenced prismatoid. A pool of feasible vertices that satisfy (2.1) is generated. The largest norm in the pool of feasible vertices is the error of the best estimate.

4. EXAMPLE

The purpose of showing an example in this section is two-fold. First, it illustrates the theorems introduced in the previous section. Second, it shows the practical aspect of the estimation procedure.

To complete the story mentioned in Section 1, one radar, ALCOR, and five optical sensors, SR1, SR3, SR5, SR6, and SR9 are the calibrated sensors. Their geometric distribution along the Kwajalein atoll is shown in Fig. 3.

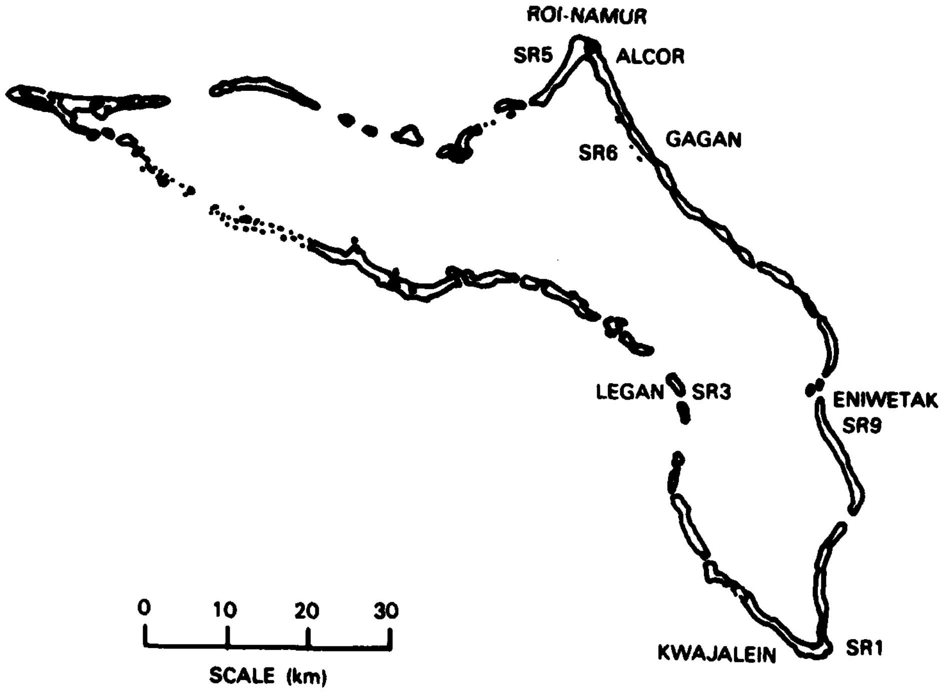

Fig. 3. The Geometric Distribution of Six Sensors.

A point northeast to the island of Noi-Namur, 90 km up, and about 240 km away is selected as the true position of the target. The a-priori information about the accuracy of each sensor is shown in Table 1. Note that the optical sensors do

TABLE 1
SENSOR ACCURACY STATEMENT

	ALCOR	SR1, 3, 5, 6, 9
Range (m)	1	100
Azimuth (μrad)	60	33
Elevation (μrad)	74	43

not measure ranges, however, an arbitrary range accuracy 100 m is chosen. The results reported in this section are insensitive to some numbers larger than 100 m.

The target position vector is transformed to a R-A-E coordinate system centered at the measurement site for each sensor. Eight possible vertices are created for each sensor according to the accuracy statement shown in Table 1. Finally, coordinate transformations are performed to transfer all vertices to a common earth-centered, earth-fixed coordinate system. Thus, six prismatoids in ths space are created, and thirty-six inequalities are put into a form given by (2.1).

Since all prismatoids have the same center (the true target position) the intersected polyhedron is symmetric. Therefore, a periodic solution exists for this artificial example. The position estimate is computed according to (3.3), and it agrees with the true position to within fractions of a meter. The errors of the estimate for various sensor combinations are shown in Table 2.

TABLE 2
ERRORS OF THE POSITION ESTIMATE

Case	Sensors	Error (m)
1	ALCOR	23
2	ALCOR + SR5	13
3	ALCOR + All SRS	13
4	One SR Alone	101
5	SR5 + SR3	77
6	SR5 + SR1	56
7	All SRS	56

For this artifical example with a specific geometry, it is concluded from Table 2 that the combination of ALCOR and one almost co-located optical sensor (Case 2) can achieve a position estimate as accurate as the estimate obtained by using all six sensors (Case 3). It is also interested to note that the accuracy of two optical sensors in Case 6 is the same as Case 7.

5. STATISTICAL INTERPRETATION OF THE EXAMPLE

It is important to note that the proposed philosophy and method should not be used to assess the quality of an unbiased estimate. If it is certain that the estimate is unbiased, a conventional covariance analysis technique is preferred. To illustrate this point of view, let us assume that the position estimate based on each of the sensor mentioned in the previous section is unbiased with equal variance. The combined variance is reduced to one sixth of the individual variance. Applying the proposed error definition we do not obtain any error reduction.

On the other hand, if it is certain that the estimate is biased and there is no way to compute the bias then the proposed error computation may be used to assess the amount of the bias of the estimate. The observability condition of an estimation problem in general is much easier to be examined than the existence of an unbiased estimate. Since we know that observability is necessary for the existence of an unbiased estimate, the proposed philosophy and method could be very valuable for assessing the quality of an estimate of an unobservable system.

6. SUMMARY AND CONCLUSION

We first explain why a non-model based estimation procedure is preferred in some occassions. We then define the best estimate and its associated error as the geometric center and the distance of the intersected error volume derived based on a number of individual sets of measurements and their own a-priori knowledge. We discover that Khachian's algorithm can be used to find the geometric center of a finite dimensional polyhedron. Once the center is located, a method is introduced to find the distance of the polyhedron. The philosophy and the technique are applied to a set of sensors located along the Kwajalein atoll for evaluating each sensor's contribution to determining the position of a target.

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